ASSIGNMENT 5 SOLUTIONS

MATH 303, FALL 2011

If you find any errors please let me know.

Manipulation

(M1) $\exists w \exists y(x = (w, y))$ *you could also expand out more if you want.*

(M2) **(1 point)** Which of the following are well formed formulas and which of the well formed ones are good?

  (a) Not well formed ($\ni$ is not in our language).

  (b) Well formed and good.

  (c) I took out too many brackets here. Let's say the formula is $(\forall x(x \in c)) \wedge (x = y)$, then this is well formed by not good.

  (d) Well formed but not good.

(M3) **(1 point)** Mark the free and bound variables in the following formulas.

  (a) $\exists x^{\text{bound2}} \exists y^{\text{bound1}}((y^{\text{bound1}} \in z^{\text{free}}) \vee (x^{\text{bound2}} \in z^{\text{free}}) \rightarrow \sim (z^{\text{free}} = w^{\text{free}}))$

  (b) $\forall z^{\text{bound1}}(x^{\text{free}} = y^{\text{free}})$

  (c) $\forall x^{\text{bound3}} \exists y^{\text{bound2}}(((x^{\text{bound3}} = y^{\text{bound2}}) \vee (y^{\text{bound2}} = z^{\text{free}})) \wedge \exists x^{\text{bound1}}(x^{\text{bound1}} \in y^{\text{bound2}}))$

(M4) (a) propositional function

  (b) propositional function

  (c) not propositional function

(M5) $\forall x \exists y((x \in y) \wedge \exists z(y \in z))$.

Pure Math

(P1) (a) Use the fact twice:

$$\exists x \exists y((y \in z) \vee (x \in z) \rightarrow \sim (z = w))$$

is equivalent to

$$\sim \forall x \sim (\exists y((y \in z) \vee (x \in z) \rightarrow \sim (z = w)))$$

which is equivalent to

$$\sim \forall x \sim (\sim \forall y \sim ((y \in z) \vee (x \in z) \rightarrow \sim (z = w)))$$

which is equivalent to

$$\sim \forall x \forall y \sim ((y \in z) \vee (x \in z) \rightarrow \sim (z = w))$$

  (b) The idea here is just to do the above to every appearance of $\exists$. Formally, let ψ be any formula in our language. Make a new formula θ which is formed as for ψ except that every time we applied rule 4 with a $\exists$ when forming ψ, instead of $\exists x A(x)$ write $\sim \forall x \sim A(x)$. Then θ and ψ are equivalent but θ has no $\exists$.

(c) Yes, $\forall x A(x)$ is equivalent to $\sim\sim \forall x \sim\sim A(x)$ which by the given fact is equivalent to $\sim \exists x \sim A(x)$. Thus we can rewrite $\forall x A(x)$ as $\sim \exists x \sim A(x)$ as in the previous part in order to convert any formula to one with no $\forall$.

(P2) Use $\uparrow$ for the Sheffer stroke, that is $A \uparrow B = \sim (A \wedge B)$.

For $\sim$ note that $A \uparrow A = \sim (A \wedge A) = \sim A$. To summarize

$$\sim A = A \uparrow A$$

For $\wedge$ note that $A \wedge B = \sim\sim (A \wedge B) = \sim (A \uparrow B) = (A \uparrow B) \uparrow (A \uparrow B)$. To summarize

$$A \wedge B = (A \uparrow B) \uparrow (A \uparrow B)$$

For $\vee$ note that $A \vee B = \sim ((\sim A) \wedge (\sim B)) = (\sim A) \uparrow (\sim B) = (A \uparrow A) \uparrow (B \uparrow B)$. To summarize

$$A \vee B = (A \uparrow A) \uparrow (B \uparrow B)$$

Ideas

(I1) (a) One possibility is

$$\begin{aligned} S_1 &: S_2 \text{ is true.} \\ S_2 &: S_3 \text{ is true.} \\ &\vdots \\ S_{n-1} &: S_n \text{ is true.} \\ S_n &: S_1 \text{ is false.} \end{aligned}$$

(b) To show that there is no consistent way to assign truth values to the sentences of Yablo's paradox, first suppose S_1 is true. Then all the remaining sentences are false, but then it is false that S_2 is false, so we have a contradiction. Now suppose S_1 is false, so there is at least one true statement among the S_k for $k > 1$. Say S_i is true. Then every statement after S_i is true, and thus S_{i+1} is false. This is again a contradiction. In both cases we got a contradiction and so there is no consistent way to assign truth values to all the statements.

For the comparison to the Liar's paradox, answers will vary.

(I2) Answers will vary.