Math 303, Fall 2011, Lecture 15

① Review for the midterm

The midterm is next class! Thursday November 3

My office hours are as usual Today October 31 after class (11:30–12:30)
and Wednesday Nov 2 2:30–3:30

Nathan will have office hours Wednesday Nov 2 9:30–11:30 in K9512.1

What is the take-home message from each lecture?

Russell's paradox and the axioms

Lecture 1: $\in$, $\subseteq$, Russell's paradox

Lecture 2: building sets: $\emptyset$, $\{A, B\}$, unions

Lecture 3: more building sets: Power set, intersections, complements
don't require new axioms
and properties of these (eg De Morgan's law)

Lecture 4: constructing things: ordered pairs and cartesian product constructions

Lecture 5: more constructing things: natural numbers, successor sets, ω ← needed a new axiom
finite and infinite.

Lecture 6: induction.

Lecture 7: even more constructing things: functions, one-to-one
onto
measuring the size of sets using one-to-one and onto functions

history

Lecture 8: history of the axioms
back to constructions: families, general cartesian products.
(why? – to get to axiom of choice)

axiom of choice

Lecture 9: } axiom of choice
Lecture 10:

logic and truth.

Lecture 11: symbols of our formal language, Syntax wff.

Lecture 12: free and bound variables, good formulas, sentences
abbreviations

Lecture 13: Propositional calculus (including truth tables)
liar's paradox

(2) Questions regarding the midterm or the work we've done so far?

P1 from A1

Let A and B be two sets formed using only $\emptyset$ and unordered pairing

say (*) $A = B$ but formed in different way

say A is the set with pairing applied a minimum number of times to get an equality like (*)

Want to find a contradiction.

Case 1 say A has 1 element. Then so does B

Then $A = \{A_1, A_1\} = \{A_1\}$

and $B = \{B_1, B_1\} = \{B_1\}$

but sets are determined by their elements

so $A_1 = B_1$ but A_1 has fewer applications of pairing

so $A_1 = B_1$ contradicts the minimality of A.

Case 2 say $A = \{A_1, A_2\}$

then B also has 2 element, say

$$B = \{B_1, B_2\}$$

Now either $A_1 = B_1$ and $A_2 = B_2$

or $A_1 = B_2$ and $A_2 = B_1$

without loss of generality $A_1 = B_1$ and $A_2 = B_2$.

Now since A and B were formed differently. Either A_1 and B_1 were formed differently or A_2 and B_2 were formed differently (or both)

But both A_1 and A_2 used fewer applications of pairing to construct them and so either $A_1 = B_1$ or $A_2 = B_2$ gives a contradiction to the minimality of A

Go over families

Say we have sets A_i indexed by $i \in I$

This is a family. To encode it in set theory use functions

$$f : \underbrace{I}_{\text{domain of } f \,=\, \text{index set}} \longrightarrow P \quad \text{where } A_i \in P$$

most often all A_i are a subset of some E

then $p = \mathcal{P}(E)$

then $f(i) = A_i$ ← this is how you encode the family as a function.

eg let $I = \omega$ let $A_i = \{ j \in \omega \mid j \text{ divides } i \}$

write this family as a function

$$f : \omega \to \mathcal{P}(\omega)$$

$$f(i) = A_i$$

make it feel more like a sequence but playing same role as A_i

eg let $I = \omega$ let $a_i = i + 3$

$$f : \omega \to \omega$$

$$f(i) = i + 3$$