Math 303, Fall 2011, Lecture 22

① Cantor diagonalization

It would be easy to wonder from last time if anything is larger than countable.

But there is

Cantor's theorem Let X be any set, then

$$X < \mathcal{P}(X)$$

proof The function $f: X \to \mathcal{P}(X)$

$f(x) = \{x\}$ is

It remains to show that

Suppose to the contrary

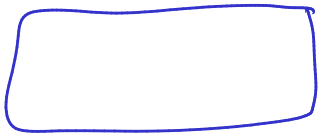
let $A = \{x \in X : x \notin g(x)\}$

Notes and consequences

(1)

② $\mathcal{P}(X) \sim 2^X$

by



③ This result is often called Cantor Diagonalization what is diagonal about it?

x	$g(x)$
x_1	$g(x_1) \subseteq X$
x_2	$g(x_2) \subseteq X$
x_3	$g(x_3) \subseteq X$
$\vdots$	

x	$g(x)$
x_1	
x_2	
x_3	
$\vdots$	

What is A?

④ One special case is

(tricky point - why did it I use binary expansions?)

In fact

② Next time

- Cardinals
- More Paradoxes and other crazy facts.