Math 303, Fall 2011, Lecture 6

① The principle of Mathematical induction

Remember ω is the smallest set $\boxed{\text{with } 0 \in \omega \text{ and } n^+ \in \omega \text{ whenever } n \in \omega}$

such a set is a successor set

We saw that

ω is a subset of every successor set.

Here's another way to write that

this is called

How does this relate to mathematical induction in the usual sense?

(see www.math.cornell.edu/~mec/2008-2009/ABjorndahl/ppmi.pdf
if you want a reminder on induction, or just some fun induction puzzles)

lets do an example. Suppose we want to prove inductively that $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$

Last time we left some properties of natural numbers to prove once we had induction

1. No natural number is a subset of any of its elements

proof

2 let n be a natural number, if $x \in n$ then $x \subseteq n$

proof

3 Let m and n be natural numbers

If $n^+ = m^+$ then $n = m$

proof

What is wrong with the following proof

Claim All horses are the same colour

proof

Where is the problem? (this is due to Pólya)

A trickier example:

The unexpected examination (hanging, tiger)

Suppose

M T W Θ F

What is wrong with the student's argument?

② Next time

Functions

finite and infinite

Please read Halmos sections 7 and 8