MATH 343, SPRING 2012, ASSIGNMENT 3

DUE THURSDAY FEBRUARY 7, 2013 IN CLASS

Do **any three** of the following four problems. If you do more than three, only the first three will be graded.

(1) Let $\mathcal{T}$ be the simple class of plane rooted trees where each vertex has a number of children which is divisible by 3. Find a formula for $[x^n]T(x)$.

(2) Show that the number of compositions of size n with exactly k summands each at most r is given by

$$[z^n]\left(z\frac{1-z^r}{1-z}\right)^k$$

and find a closed formula for this number (as a function of n, k, r). (Your answer should be a finite sum.)

(3) Let $\mathcal{T}$ be the class of rooted trees with red and blue vertices where every red vertex has an even number of blue children and no red children, and every blue vertex has at most 2 blue children and any number of red children.
   (a) Give a specification for $\mathcal{T}$ (it will probably be a system)
   (b) Using a program/package like combstruct for Maple answer the following questions.
      (i) What is t_{300}?
      (ii) Estimate what fraction of elements of $\mathcal{T}_{300}$ have a blue root by generating some random trees in $\mathcal{T}_{300}$.

(4) A permutation of size n is a bijection of $\{1, 2, \ldots, n\}$ to itself. Let

$$\sigma : \{1, 2, \ldots, n\} \to \{1, 2, \ldots, n\}$$

be a permutation. One way to represent σ is as the list of its values

$$(\sigma(1), \sigma(2), \ldots, \sigma(n))$$

Consider the order on permutations given by lexicographic order on the lists of values.
   (a) Write an algorithm SuccessorPerm which takes n and a permutation σ of size n and returns the next permutation in the order described above. You can either write a program in a language of your choice or write in pseudocode.
   (b) Test your answer to the previous part on a few examples (if you used pseudocode you will need to step though by hand, otherwise submit the output of your program)
   (c) What is the worst case runtime of your algorithm?