MATH 343, SPRING 2012, ASSIGNMENT 5

DUE THURSDAY MARCH 7, 2013 IN CLASS

Do **any three** of the following four problems. If you do more than three, only the first three will be graded.

(1) Let $\mathcal{T}$ be the class of trees with red and blue vertices where each red vertex has at most one red child and any number of blue children, and each blue vertex has an even number of children (with no restriction on colour).
   (a) Give a specification for $\mathcal{T}$
   (b) Implement a recursive generator (pure recursive, not Boltzmann) for $\mathcal{T}$.
   (c) Generate two trees from $\mathcal{T}$ of size $n = 25$ and draw a picture of each of them. Comment on their qualitative shape – are they short or tall, fat or thin, are they similar to each other, etc. (If you write code for the picture drawing please feel welcome to use a larger value of n; 25 was chosen so that people who draw them by hand don't go completely insane.)

(2) This question is about combinatorial classes $\mathcal{C}$ where $C(x)$ has radius of convergence $\rho > 0$.
   (a) Find an example of such a combinatorial class where $\lim_{x\to\rho^-} C(x) = \infty$. Justify.
   (b) Find an example of such a combinatorial class where $\lim_{x\to\rho^-} C(x)$ is finite. Justify.

(3) (a) For each of the following classes plot the expected size of a Boltzmann model with parameter x as a function of x.
   - plane trees
   - binary words with all blocks of 0s of even length.
   - a combinatorial class with generating function $P(x) = e^{e^x-1}$ (this is the exponential generating function for set partitions, but that's another story)

   (b) The expected value only tells part of the story – we also want to know whether the distribution is more spread out or more concentrated. The *variance* is one measure of this and in our context for a generating function $A(x)$ is given by

$$V_x = \frac{x^2A''(x) + xA'(x)}{A(x)} - \frac{x^2A'(x)^2}{A(x)^2}.$$

   For each of the combinatorial classes from the previous part plot the variance of the Boltzmann model as a function of x.
   (c) Which combinatorial class from the first part do you think will work best with a Boltzmann generator? Which will be worst?

(4) Implement a Boltzmann generator for binary words with no 00 subword. What is the maximum value for x? Find the value of x such that the expected word size is 100. Run the algorithm with this value of x until you get a tree in the range 80 - 120. How many times did you have to run it?