Math 343 lecture 15

① Minimal change orderings for k-subsets

What is the distance between two subsets of $\{1, \ldots, n\}$
view them as binary words and take the Hamming distance

equivalently

Def Let $S, T \subseteq \{1, \ldots, n\}$
$d(S, T) =$

The minimum distance between two distinct k-element subsets of $\{1, \ldots, n\}$ is

revolving door order

One way we can think of the reflected binary Gray code construction is as follows

-

-

Now lets do the same thing for k-subsets

Write $R(n,k)$ for our desired order on k subsets of $\{1, \ldots, n\}$

Recall

We will accomplish the first bullet point by using a decomposition which mirrors (*)

Namely

So we can define

Def $R(n,n) =$, $R(n,0) =$

$R(n,k) =$.

$0 \leq k \leq n$, $n > 0$

eg $R(2,1) =$

eg $R(3,1) =$

eg $R(3,2) =$

eg $R(4,1) =$

eg $R(4,2) =$

eg $R(4,3) =$

We want to prove this really gives a minimal change ordering. The first step is to characterize the first and last elements

Prop $R(n,k)_0 =$ and $R(n,k)_{\binom{n}{k}-1} =$ for $k>0$

proof by induction on n

let $n=1$, then $k=1$ and $R(1,1) = (\{1\})$
so the result holds

Assume the result holds for $n-1$ (and all $0 \leq k \leq n-1$)

Take $0 \leq k \leq n$ and consider $R(n,k)$

If $k=n$

So assume $k<n$, then

Prop Let $n > 0$ and $0 \leq k \leq n$ be integers. Then $R(n,k)$ is a minimal change ordering on the set of k-subsets of $\{1, 2, \ldots, n\}$

proof by induction on n

If $n = 1$ then $k = 1$ and the result is true

Assume the result holds for $n-1$ (and all $k \leq n-1$) and now take $0 \leq k \leq n$ and consider $R(n,k)$

If $k = 0$ or $k = n$ then

Suppose $1 \leq k \leq n-1$

As with the binary reflected Gray code this order is cyclic which we can check by continuing the above calculation

(2) Next time

Working out the successor algorithm for revolving door

Minimal change orderings for permutations