Math 343, Lecture 17

(1) Generating trees

One last approach for exhaustive generation is generating trees
lets look at permutations as an example.

As for Trotter-Johnson order we can form a permutation of order n by

Lets organize this in a tree (1)

If we cut off the tree at a given level then

The shape of the tree is

The general feature along these lines that we are going to want is

Formally

Def A generating tree is

eg for permutations the rule is

The point is that

so

or

Here's another example

Def an involution is a permutation which is its own inverse that is

What do involutions look like — to see it lets draw a permutation a different way

eg $(1\,4\,2\,3\,5)$

Is this an involution

No

eg

What can the graph of an involution look like?

So an involution involves only fixed points and swaps

This tells us a way to make a generating tree:

(1)

what are the labelling rules?

So the rules are

One last example

Def A set partition of $X = \{1, 2, \ldots, n\}$ is

parts

eg the set partitions of $\{1,2,3\}$ are

We can make a set partition of $\{1,\dots,n+1\}$ out of a set partition of $\{1,\dots,n\}$ by

The label of a vertex is

ε

② Next time

Knapsack problem