Math 343 Lecture 3

(1) Partial fractions

You may recall partial fractions from calculus (where its used to integrate rational functions) or maybe from MACM 201 for coefficient extraction which is what we'll do today

Proposition Let f_1, f_2, g be polynomials ( )
such that ( )
and

Then we can find g_1 and g_2 polynomials with

and

Note by convention the degree of the 0 polynomial is $-\infty$

proof Since f_1 and f_2 have no common factors

by polynomial division we can also write

let $g_2 =$

It just remains to check

Suppose for a contradiction

Theorem (partial fraction decomposition)

Let f and g be polynomials and write $f = f_1^{a_1} \cdots f_r^{a_r}$

where

Suppose then

for

proof By the proposition

So consider

eg Rewrite $\dfrac{1+x}{(1-x)^2(1+2x)}$ using partial fractions decomposition

How to find A, B, C?

$\left(\frac{2}{9}, \frac{1}{9}, \frac{2}{3}\right)$

② Partial fractions as an algorithm

Now that we know partial fraction decomposition exists how would we implement it?

So far we have 2 ideas

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The system of equations is

One can do better with Strassen's algorithm (and newer improvements) which let you multiply two $n \times n$ matrices in $O(n^{2.81})$ operations (current best $O(n^{2.3727})$)
The algorithms also let you invert matrices in the same time and hence solve systems

However

The Euclidean algorithm approach is better. With this idea one can get to $O(\log n \cdot M(n))$ where $M(n)$ is the runtime to multiply two polynomials of degree n.

but

for a runtime of $O(n \log^2 n)$ for partial fractions (the details of all this are unfortunately beyond this course)

But what if f is not given to us factored?

3 Coefficient extraction using partial fractions

To find $[x^n] \frac{g(x)}{f(x)}$:

1

2

3

4

eg find $[x^n] \frac{x+1}{x^2+2x-3}$

1 and 2 This one is pretty easy but lets use maple just to see how to do it

convert(R, parfrac)

3

4

4 Next time

Basic combinatorial constructions

please read the notes.