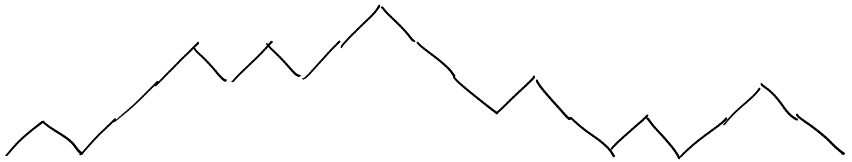
Math 343, Lecture 9

(1) Ranking and unranking Dyck paths

Recall Dyck paths



We can think of a Dyck path as

So we have a map

eg

Which binary strings do we get?

Answer those which are totally balanced

Def A binary string w of length $2n$ is totally balanced if

- .
- .
- .

Translating back to up and down steps

Viewing Dyck paths as binary words we can put them in lexicographic order

As with k-subsets the key to ranking and unranking will be good formulas for the paths preceeding a given path in the order. To this end define:

Def Let $\mathcal{M}_{2n}(x,y)$ be

Let $m_{2n}(x,y) = |\mathcal{M}_{2n}(x,y)|$

prop Let x, y, n be positive integers with $x+y$ even and $x+y \leq 2n$ then

$$m_{2n}(x,y) = \binom{2n-x}{n-\frac{x+y}{2}} - \binom{2n-x}{n-1-\frac{x+y}{2}}$$

proof If we don't worry about crossing the x axis or not

Now we just need to subtract off the ones which go below the x axis.

Given such a walk

Every walk from $(x, -y-2)$ to $(2n, 0)$ is obtained exactly once in this way

Finally count the walks from $(x, -y-2)$ to $(2n, 0)$

So

Now let $w = w_1 w_2 \dots w_{2n}$ be a word of length $2n$ corresponding to a Dyck path

Consider

This gives the algorithm

Algorithm RankDyck

input: n, w w a totally balanced word of length $2n$

$y = 0$

$r = 0$

for

output:

and

Algorithm UnrankDyck

input: n, r

$y = 0$

$rnew = 0$

for

examples: tutorial

output

② Next live Prüfer correspondence