ASSIGNMENT 5

MATH 303, FALL 2011

Instructions: *Do at least* ***3 points from each section*** *and at least* ***10 points total****. Up to 12 points will be graded, but your maximum score is 10. If you hand in more than 12 points please indicate which ones you want graded, otherwise the first 12 will be graded.*

Manipulation

(M1) **(1 point)** Write a formula in our formal language which says "x is an ordered pair". You may use abbreviations we discussed in class.

(M2) **(1 point)** Which of the following are well formed formulas and which of the well formed ones are good?
- (a) $x \ni y$
- (b) $(x \in c) \wedge (c = c)$
- (c) $\forall x(x \in c) \wedge (x = y)$
- (d) $\forall z(x \in c)$

(M3) **(1 point)** Mark the free and bound variables in the following formulas.
- (a) $\exists x \exists y((y \in z) \vee (x \in z) \rightarrow \sim (z = w))$
- (b) $\forall z(x = y)$
- (c) $\forall x \exists y(((x = y) \vee (y = z)) \wedge \exists x(x \in y))$

(M4) **(1 point)** Which of the following are propositional functions in the variables A_1, A_2, A_3, and A_4?
- (a) $A_2 \wedge (\sim A_4)$
- (b) A_3
- (c) $A_3 = A_1$

(M5) **(1 point)** Write a good formula in our formal language which says the same thing as

$$\forall x \exists y((x \in y) \wedge \exists x(y \in x))$$

Pure Math

(P1) **(3 points)** Let $A(x)$ be a formula with x as a free variable. We can express $\exists x A(x)$ as $\sim \forall x(\sim A(x))$. (Convince yourself that this is true, but you don't need to write anything down about it.)
- (a) Use that fact to rewrite $\exists x \exists y((y \in z) \vee (x \in z) \rightarrow \sim (z = w))$ so that it does not involve $\exists$.
- (b) Prove that you can rewrite any formula in our language so that it does not involve $\exists$.
- (c) Can you rewrite any formula in our language so that it does not involve $\forall$ (it can use $\exists$)?

(P2) **(4 points)** Express $\sim$, $\vee$, and $\wedge$ in terms of the Sheffer stroke (NAND).

Ideas

(I1) **(3 points)**

(a) We have a version of the Liar's paradox with one sentence and one with two sentences. Can you give one with any finite number of sentences?

(b) Stephen Yablo in 1993 proposed the following paradox

S_1: For all $k > 1$, S_k is false
S_2: For all $k > 2$, S_k is false
$\vdots$
S_i: For all $k > i$, S_k is false
$\vdots$

Explain how there is no consistent way to assign truth to each of these sentences and compare this to the liar's paradox.

References:

- "Paradox Without Self-Reference". Analysis 53 (4): 251–252. 1993.
- `http://en.wikipedia.org/wiki/Yablo%27s_paradox`

(I2) **(3 points)** Plato, on a number of occasions, ascribes to Socrates some variant of the statement "I know only that I know nothing". Compare this to the Liar's paradox.