Math 303, Fall 2011, Lecture 3

(1) Power sets

We've been considering the subsets of a set E. Do all such subsets form a set?

Axiom of Power sets

For every set E, there is a set $\mathcal{P}$ consisting of precisely the subsets of E

that is $A \subset E$ if and only if $A \in \mathcal{P}$

We write $\mathcal{P}(E)$ for the power set of E

eg what is $\mathcal{P}(\{1,2,3\})$?

eg what is $\mathcal{P}(\{a\})$?

eg what is $\mathcal{P}(\emptyset)$?

Question. If E has n elements how many elements does $\mathcal{P}(E)$ have?

(2) Constructing things part 1 — intersections and complements.

These sets and axioms are ok, but if we need a new axiom for each new construction that isn't very good.

Fortunately we can already do stuff with what we have

Define $A \cap B = \{$

eg $\{1, 2, 3, 4\} \cap \{2, 4, 6\} =$

eg $\{\{\phi\}, \phi\} \cap \phi =$

but wait that definition is no good at all!

Fix

but

Properties

① What is $A \cap \emptyset$?

② What is $A \cap A$?

③ What is $A \cap (B \cup C)$?

that's a little harder

Suppose $x \in A \cap (B \cup C)$

This shows $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

Likewise if $x \in (A \cup B) \cup (A \cap C)$

then

So $x \in A \cap (B \cup C)$

Therefore $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

This should remind you of

But for sets it is also true that

(4) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(but not !)

can you prove it in the same way?

More properties

(5) $A \cap B =$

(6) $A \cap (B \cap C) =$

What about extending intersections to sets of sets the way we did for unions

Define For any nonempty set $\mathcal{C}$

$$\bigcap \mathcal{C} = \bigcap_{B \in \mathcal{C}} B \quad =$$

eg $\mathcal{C} = \{ \{a, b\}, \{b, c\}, \{a, b, c\} \}$

$$\bigcap \mathcal{C} =$$

$\boxed{\text{Question for the break} \quad \text{What is } \bigcap \emptyset \text{ ?}}$

$\cap \emptyset$ should be

The complement

Let A and B be sets then the set difference or relative complement of B in A is

$$A \setminus B = \{ x \in A : x \notin B \}$$

eg $A =$ $B =$

$A - B =$

eg $A =$ $B =$

$A - B =$ $\leftarrow$

eg $A \setminus A =$

Sometimes we will be taking a lot of complements in the same outer set. Then the following notation is more convenient

$$\boxed{\text{Fix a set } E. \quad \text{Let } A' = E \setminus A, \text{ the complement of } A \text{ (in } E)}$$

eg $E =$, $A =$

then $A' =$

eg

Try these. Fix a set E

1. What is $(A')'$

2. What is $\emptyset'$

3. What is E'

4. What is $A \cap A'$

5. What is $A \cup A'$

Here are some more

(6) $A \subseteq B$ if and only if $B' \subseteq A'$

Suppose $A \subseteq B$.

For the other direction

(7) De Morgan's Laws

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

Suppose $x \in (A \cup B)'$ So $x \in E$ and $x \notin A \cup B$

Conclusion: $(A \cup B)' \subseteq A' \cap B'$

Now suppose $x \in A' \cap B'$.

Similarly for the other one.

③ Next time

ordered pairs and cartesian products

$\boxed{\text{please read Halmos section 6}}$