MATH 343, SPRING 2012, ASSIGNMENT 2

DUE THURSDAY JANUARY 24, 2013 IN CLASS

Do **any three** of the following four problems. If you do more than three, only the first three will be graded.

(1) (a) Let $\mathcal{A} = \{00, 1, 01\}$. Are the words of $\textsc{Seq}(\mathcal{A})$ uniquely generated? Either give a word that can be generated in more than one way, or give a proof that each word in the class can be built in only one way.
   (b) Let $\mathcal{B} = \{00, 101, 11\}$. Are the words of $\textsc{Seq}(\mathcal{B})$ uniquely generated? Either give a word that can be generated in more than one way, or give a proof that each word in the class can be built in only one way.

(2) This question is about strings using the letters $\{0, 1, 2\}$ and counted by length. Let $\mathcal{C}$ be the combinatorial class of such strings with no consecutive 2s.
   (a) Find $C(x)$.
   (b) Find a formula for c_n by partial fractions.

(3) Let $\mathcal{W}^{(k)}$ be the class of binary strings counted by length, with no k or more consecutive 0s.
   (a) Show that the generating function for $\mathcal{W}^{(k)}$ is
$$W^{(k)}(x) = \frac{1 - x^k}{1 - 2x + x^{k+1}}.$$
   (b) Find $W^7_{200} - W^6_{200}$ (using a computer is fine) Explain why this is the number of binary strings of length 200 with *at least one* block of 0s of length exactly 6.
   (c) Use the previous part to determine the probability that a binary string of length 200 has at least one block of 0s of length exactly 6. If you flip a fair coin 200 times should you be surprised if you get 6 heads in a row?

(4) Let $\mathcal{D}$ be the class of all plane rooted trees where each vertex has either 0 children or at least 2 children.
   (a) Give a combinatorial specification for $\mathcal{D}$, that is, give an equation involving $\mathcal{D}$ using operators like $\times, +, \textsc{Seq}$, and classes $\mathcal{E}, \mathcal{Z}$.
   (b) Give a polynomial equation involving $D(x)$.